1. If $y = x\sin x$, then $\frac{dy}{dx} =$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

$u = x \qquad u' = 1$

$v = \sin x \qquad v' = \cos x$

$x\cos x + \sin x$

B

---

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

increasing → slope is positive

$f(x) = 300x - x^3$

$f'(x) = 300 - 3x^2$

$300 - 3x^2 = 0 \qquad 3 \quad 100 - x^2$

$x^2 = 100$

$x = \pm 10$

+ + +

− + −

⊖ −10 ⊕ +10 ⊖

$[-10, 10]$

B

GO ON TO THE NEXT PAGE.

3. $\int \sec x \tan x \, dx =$ Secx +C

Property must be memorized!

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\frac{\sec^2 x}{2} + C$

(D) $\frac{\tan^2 x}{2} + C$

(E) $\frac{\sec^2 x \tan^2 x}{2} + C$

A

---

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

$f(x) = 7x - 3 + \ln(x)$

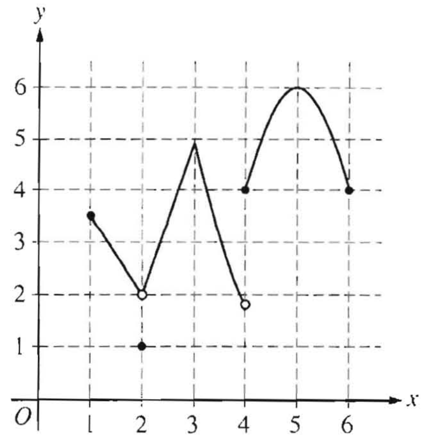
$f'(x) = 7 + \frac{1}{x}$

$f'(1) = 7 + \frac{1}{1}$

$= 8$

E

GO ON TO THE NEXT PAGE.



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists. $\lim_{x \to 2^-} = 2$ $\lim_{x \to 2^+} = 2$

(B) $\lim_{x \to 3} f(x)$ exists. $\lim_{x \to 3^-} = 5$ $\lim_{x \to 3^+} = 5$

(C) $\lim_{x \to 4} f(x)$ exists. $\lim_{x \to 4^-} = 1.9$ $\lim_{x \to 4^+} = 4 \rightarrow \lim_{x \to 4} = \text{DNE}$

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

C

---

6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

$v(t) = 6t - t^2$

$s(t) = 3t^2 - \frac{1}{3}t^3 + C$

$C = 0$

$t = 0 \rightarrow s = 0$

$t = 3 \rightarrow s = 18$

D

GO ON TO THE NEXT PAGE.

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

Chain Rule

$= 5(x^3 - \cos x)^4 * (3x^2 + \sin x)$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

E

---

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$\square_1 = 3(6.2) = 18.6$

$\square_2 = 5(5.9) = 29.5$

$\square_3 = 3(5.6) = 16.8$

50.0 ← initial amount
18.6
29.5
16.8
114.9

$6.2 \times 3 = 18.6$

$5.9 \times 5 = 29.5$

$5.6 \times 3 = 16.8$

C

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

E

$$\frac{(2x+1)\cancel{(x-2)}}{\cancel{(x-2)}} = k$$

$$2x + 1 = k$$
$$2(2) + 1 = k$$
$$5 = k$$

---

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

$$\int_0^2 e^{\frac{1}{2}x}\,dx$$

$u = \frac{1}{2}x$
$du = \frac{1}{2}dx$
$2du = dx$

$x = 0 \rightarrow u = 0$
$x = 2 \rightarrow u = 1$

$$2\int_0^1 e^u\,du$$
$$= 2\left[e^u\right]_0^1$$
$$= 2\left[e^1 - \cancel{e^0}^{\,1}\right]$$
$$2e - 2$$

A

GO ON TO THE NEXT PAGE.

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

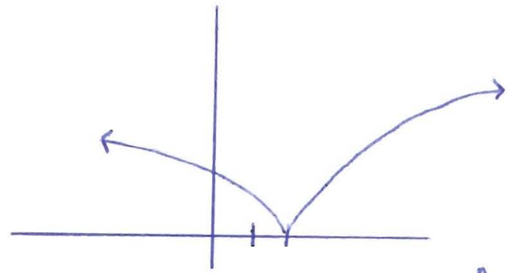
(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

x	y
-2	2
0	$\sqrt{2}$
2	0
6	2



A

---

12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\, du$ (B) $2\int_1^{4} e^u\, du$ (C) $2\int_1^{2} e^u\, du$ (D) $\frac{1}{2}\int_1^{2} e^u\, du$ (E) $\int_1^{4} e^u\, du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

$x = 1 \rightarrow u = 1$

$x = 4 \rightarrow u = 2$

$2du = \frac{1}{\sqrt{x}}\, dx$

$2\int_1^2 e^u\, du$

C

GO ON TO THE NEXT PAGE.

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

D

$$\int_1^3 2dx + \int_3^5 x-1\,dx = [6-2] + \left[\left(\tfrac{25}{2}-5\right) - \left(\tfrac{9}{2}-3\right)\right]$$

$$= 2x\Big|_1^3 + \left[\tfrac{1}{2}x^2 - x\right]_3^5 \qquad \left(\tfrac{25}{2}-\tfrac{10}{2}\right) - \left(\tfrac{9}{2}-\tfrac{6}{2}\right)$$

$$= 4 + \tfrac{15}{2} - \tfrac{3}{2}$$

$$\tfrac{8}{2} + \tfrac{15}{2} - \tfrac{3}{2}$$

$$= 10$$

---

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

$$\sqrt{(3x-2)^2 - 4} \qquad (9x^2 - 12x)^{1/2}$$

$$(3x-2)(3x-2)$$

$$\sqrt{9x^2 - 12x \cancel{+4} \cancel{-4}}$$

$$= \tfrac{1}{2}(9x^2-12x)^{-1/2} + (18x - 12)$$

A

$$= \frac{\cancel{18}^{9}x - \cancel{12}^{6}}{\cancel{2}\sqrt{9x^2-12x}} = \frac{9x-6}{\sqrt{9x^2-12x}}\Bigg|_{x=3}$$

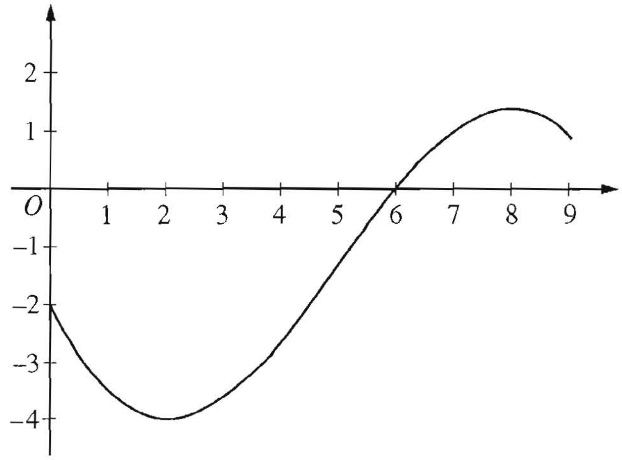
$$= \frac{27-6}{\sqrt{81-36}} = \frac{21}{\sqrt{45}} = \frac{21}{3\sqrt{5}} = \frac{7}{\sqrt{5}}$$

$$\sqrt{45} = 3\sqrt{5}$$

9 5

$$\begin{array}{r} \overset{7}{\cancel{8}}11 \\ -36 \\ \hline 45 \end{array}$$

GO ON TO THE NEXT PAGE.



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

$h(6) = \int_0^6 f(t)dt \rightarrow$ negative

$h'(6) \Longrightarrow h'(x) = \frac{d}{dx}\int_0^x f(t)dt = f(x) = f(6) = 0$

$h''(6) \rightarrow h'(x) = f(x)$

$h''(x) = f'(x)$

$h''(6) = f'(6)$ POSITIVE

slope at 6.

A

$h(6) < h'(6) < h''(6)$

GO ON TO THE NEXT PAGE.

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \dfrac{a + b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$

velocity = 0

$x(t) = (t-a)(t-b)$

$u = t - a \qquad u' = 1$

$v = t - b \qquad v' = 1$

B

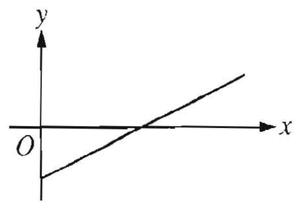
$(t-a) + (t-b) = 0$

$2t - a - b = 0$

$2t = a + b$

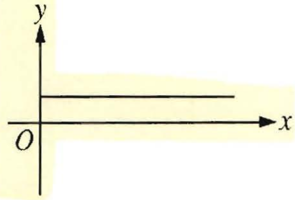
$t = \dfrac{a+b}{2}$

GO ON TO THE NEXT PAGE.



Graph of f

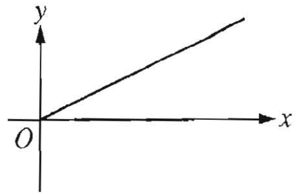
17. The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\,dt$, which of the following could be the graph of $y = g(x)$?

(A)


$$\frac{d}{dx}\int_2^x g(t)dt = g(x)$$

(B)

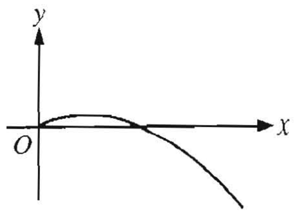

$$\rightarrow \underbrace{f'(x)}_{\text{derivative of } f.} = g(x)$$

(C)


A

f has a constant Positiive slope

(D)


(E)


GO ON TO THE NEXT PAGE.

18. $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

definition of
a derivative

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

B

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}\Big|_{x=4} = \frac{1}{4}$

---

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

$u = x$ $u' = 1$

$v = x+2$ $v' = 1$

$\frac{\cancel{x} + 2 - \cancel{x}}{(x+2)^2} = \frac{1}{2}$

cross multiply

C

$(x+2)^2 = 4$

$x + 2 = \pm 2$

$x = 0$

$x = -4$

GO ON TO THE NEXT PAGE.

20. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

$f(0)=1 \rightarrow (0,1) \xrightarrow{f^{-1}} (1,0) \rightarrow g(1)=0$

$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)} = \frac{1}{6}$

$f(x) = (2x+1)^3$

$f'(x) = 3(2x+1)^2 * 2$

$f'(0) = 3(1) * 2$

$= 6$

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2 - x}{1+4x^2}$

H.A: $y = -5$

HA: $\frac{20x^2}{4x^2} = 5$

GO ON TO THE NEXT PAGE.

22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

$u = \ln(x) \quad u' = 1/x$

$v = x \quad v' = 1$

$$\frac{x\left(\frac{1}{\cancel{x}}\right) - \ln(x)}{x^2}$$

$1 - \ln(x) = 0$

$\ln(x) = 1$

$x = e$

$x^2 = 0$

$x = 0$

$\frac{1 - \ln(x)}{x^2}$ + − ; + + + ; 0 e

max @ $x = e$

$y = \frac{\ln e}{e} = \frac{1}{e}$

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

the only function that can be integrated to a linear function is a constant.

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

Critical Point $\rightarrow$ $f'(x) = 0$ or $f'(x) = \text{DNE}$

$g(x) = x^2 e^{kx}$

$u = x^2$ $u' = 2x$

$v = e^{kx}$ $v' = ke^{kx}$

$g'(x) = 2xe^{kx} + x^2 k e^{kx}$

$xe^{kx}(2 + xe^{kx}) = 0$

$xe^{kx} = 0$

$\frac{2}{3} e^{2/3 k} = 0$

$e^{2/3 k} = 0$

$2/3 k = \cancel{\ln}(0)$

$k = 3/2$

$2 + 2/3 e^{k 2/3} = 0$

$2/3\, e^{k 2/3} = -2$

$e^{k 2/3} = -3$

GO ON TO THE NEXT PAGE.

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

$\int dy = \int 2\sin x \, dx$

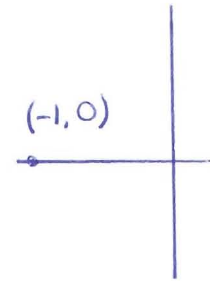
$Y = -2\cos x + C$

$1 = -2\cos(\pi) + C$

-1

$1 = 2 + C$

$C = -1$



$Y = -2\cos x - 1$

GO ON TO THE NEXT PAGE.

26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

$g'(x) = \int_0^x e^{-t^2}\,dt$

$\int_0^1 e^{-t^2}dt$ is $+$ $\rightarrow$ increasing

0 2

$g''(x) = \frac{d}{dx}\int_0^x e^{-t^2} = e^{-x^2}$

$e^{-(1)^2} = e^{-1} = \frac{1}{e}$ $+$ $\rightarrow$ concave up

0 2

27. If $(x+2y)\cdot\frac{dy}{dx}=2x-y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

$$(x+2y)\frac{dy}{dx}=2x-y$$

$$u=2x-y \qquad u'=2-\frac{dy}{dx}$$

$$v=x+2y \qquad v'=1+2\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{2x-y}{x+2y}\Big|_{(3,0)}=2$$

$$\frac{d^2y}{dx^2}=\frac{(x+2y)(2-\frac{dy}{dx})-(2x-y)(1+2\,dy/dx)}{(x+2y)^2}\Bigg|_{(3,0)}$$

$$=\frac{\overset{0}{\cancel{(3+0)(2-2)}}-(6-0)(1+2(2))}{(3+0)^2}$$

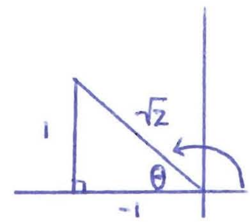
$$=\frac{-30}{9}=\frac{-10}{3}$$

GO ON TO THE NEXT PAGE.

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

$v(t) = \cos t + \sin t$ $\cos t + \sin t = 0$ $\cos t = -\sin t$



$\frac{-1}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$a(t) = -\sin t + \cos t$

$\theta = \frac{3\pi}{4}$

$-\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right)$

$-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{-2}{\sqrt{2}} = \frac{-\cancel{2}\sqrt{2}}{\cancel{2}} = -\sqrt{2}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

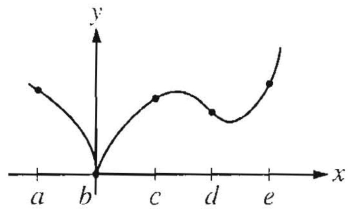
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?

(A) a (B) b (C) c (D) d (E) e

$f'(x)$ positive

$f'(x)$ increasing $\rightarrow$ $f''(x)$ positive

GO ON TO THE NEXT PAGE.

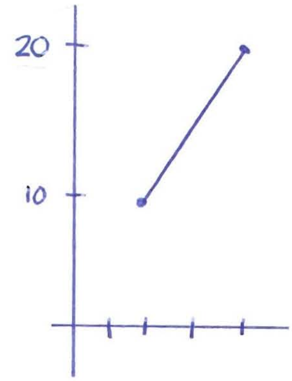
77. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.



if the function is continuous there has to be a y-value between [10, 20].

---

78. The graph of $y = e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

$y = e^{\tan x} - 2$

$y' = e^{\tan x} * \sec^2 x$

y crosses at x = .6061193

MATH 8

m = 2.961

79. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

(A) $\dfrac{a(8) - a(0)}{8}$

(B) $\dfrac{1}{8}\int_0^8 v(t)\,dt$

(C) $\dfrac{1}{8}\int_0^8 |v(t)|\,dt$

(D) $\dfrac{1}{2}\int_0^8 v(t)\,dt$

(E) $\dfrac{v(0) + v(8)}{2}$

$\dfrac{1}{b-a}\int_a^b \underbrace{f(x)}\,dx$

This equation will be the function you are trying to find the average of.

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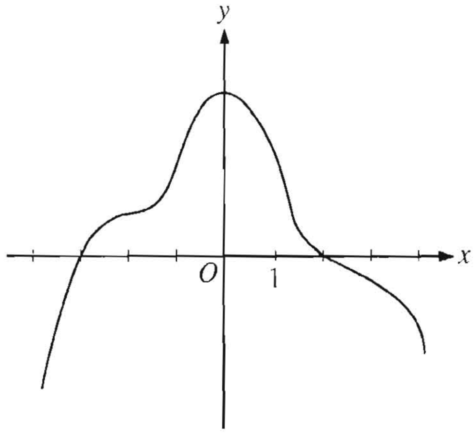


Graph of f'

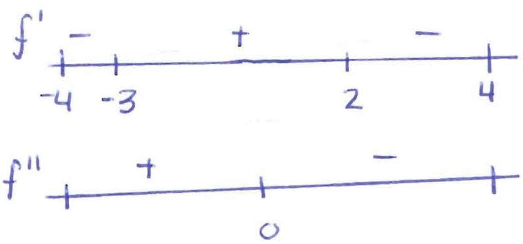
80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$. TRUE

II. The graph of f has a point of inflection at $x = -2$. FALSE

III. The graph of f is concave down for $0 < x < 4$. TRUE

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only



81. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons

(B) 420 gallons

(C) 829 gallons

(D) 1220 gallons

(E) 1376 gallons

$\int_0^{20} r(t)dt + 800 = 1220$ gallons

---

82. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$

(A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

where $f'(x) > 0 \longrightarrow f'(x) < 0$

$x = .350$

(below $\leftarrow$ above)

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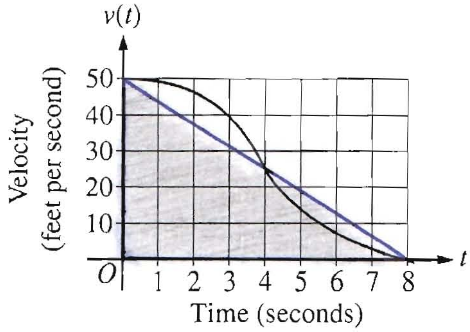

83. The graph above gives the velocity, v, in ft/sec, of a car for $0 \le t \le 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

(A) 21 ft (B) 26 ft (C) 180 ft (D) 210 ft (E) 260 ft

$\frac{1}{2}(8)(50)$

$4(50) \approx 200$ ft.

---

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

$$\underbrace{f''(x) = e^{(x^4 - 2x^2 + 1)} * (4x^3 - 4x) < 0}_{\text{below x-axis}}$$

$(-1.5, -1) \cup (0, 1)$

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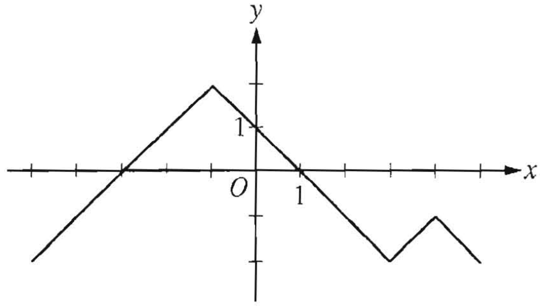


Graph of f'

85. The graph of f', the derivative of f, is shown in the figure above. The function f has a local maximum at $x =$

(A) -3 (B) -1 (C) 1 (D) 3 (E) 4

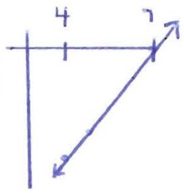
$f'(x) = 0 \longrightarrow$ @ $x = -3, 1$

max $\rightarrow$ (above to below) $x = 1$
min $\rightarrow$ (below to above) $x = -3$

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)\,dt = 0$, which of the following could be a table of values for the function f ?

(A)

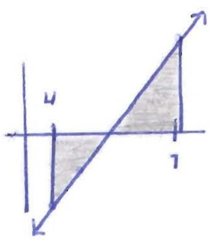
x	$f(x)$
4	-4
5	-3
7	0



slope is positive
area from 4-7 is zero

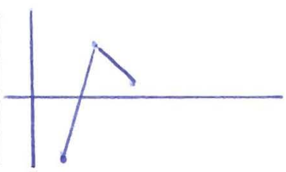
(B)

x	$f(x)$
4	-4
5	-2
7	5



(C)

x	$f(x)$
4	-4
5	6
7	3

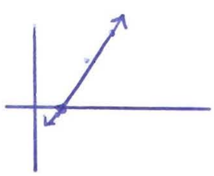


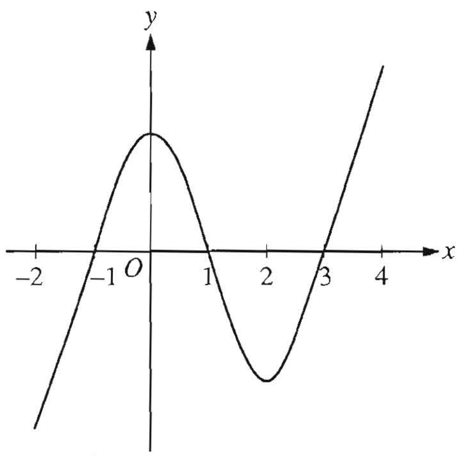
(D)

x	$f(x)$
4	0
5	0
7	0

(E)

x	$f(x)$
4	0
5	4
7	6





Graph of f''

87. The graph of f'', the second derivative of f, is shown above for $-2 \le x \le 4$. What are all intervals on which the graph of the function f is concave down?

below x-axis

(A) $-1 < x < 1$

(B) $0 < x < 2$

(C) $1 < x < 3$ only

(D) $-2 < x < -1$ only

(E) $-2 < x < -1$ and $1 < x < 3$

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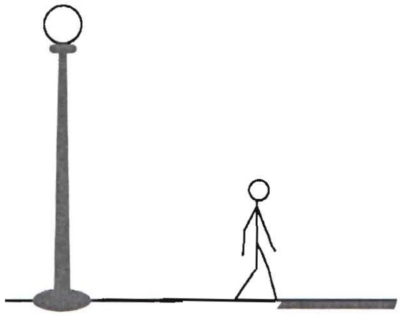


$$\frac{15}{x+s} = \frac{6}{s}$$

$$6x + 6s = 15s$$

$$6x = 9s$$

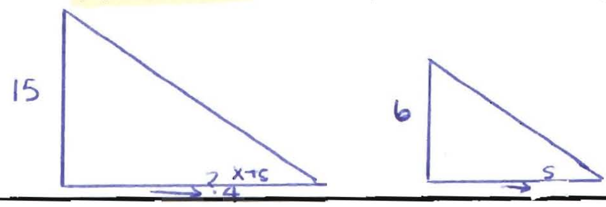
$$6\,dx/dt = 9\,ds/dt$$

$$\frac{dx}{dt} = \frac{6(4)}{9} = \frac{24}{9}$$

$$\approx 2.667$$

88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec



89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

$$v(3) - \underbrace{v(0)}_{=5} = \int_0^3 a(t)\,dt$$

$$v(3) = \int_0^3 a(t)\,dt + 5$$

$$= 11.710$$

90. Let f be a function such that $\int_{6}^{12} f(2x)\,dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t)\,dt = 5$

(B) $\int_{12}^{24} f(t)\,dt = 20$

(C) $\int_{6}^{12} f(t)\,dt = 5$

(D) $\int_{6}^{12} f(t)\,dt = 20$

(E) $\int_{3}^{6} f(t)\,dt = 5$

$u = 2x$ $\quad x = 6 \rightarrow u = 12$

$du = 2dx$ $\quad x = 12 \rightarrow u = 24$

$\frac{1}{2}du = dx$

$$\frac{1}{2}\int_{12}^{24} f(u)\,du = 10$$

$$\int_{12}^{24} f(u)\,du = 20$$

x	-2	0	3	5	6
$f'(x)$	3	1	4	7	5

91. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of f is concave up.

(B) The graph of f has at least two points of inflection.

(C) f is increasing.

(D) f has no critical points.

(E) f has at least two relative extrema.

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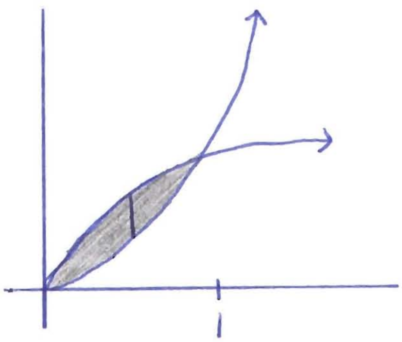
92. Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph of $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x-axis are squares. What is the volume of the solid?

(A) 0.129 (B) 0.300 (C) 0.333 (D) 0.700 (E) 1.271



$\int_0^1 (\sqrt{x} - x^2)^2 \, dx = .129 \ u^3$

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

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t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$w'(12) \approx \frac{67.9-61.8}{15-9} = \frac{6.1}{6} \text{ °F/min}$$

At approx. 12 minutes,
The temperature of the water is increasing at a rate of 1.017 °F per minute.

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(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} w'(t)\,dt \approx w(20) - w(0) = 71-55 = 16 \text{ °F}$$

The water has warmed 16°F in t=0 to t=20 minutes

Continue problem 1 on page 5.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$\square_1 = 4\,(55) = 220$

$\square_2 = 5\,(57.1) = 285.5$

$\square_3 = 6\,(61.8) = 370.8$

$\square_4 = 5\,(67.9) = 339.5$

1215.8

avg. value $= \frac{1}{20}\left[1215.8\right] = 60.79$

$W'(t)$ is always positive

↳ a Left Riemann sum is an under approximation

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$W(25) - \underbrace{W(20)}_{=71} = \int_{20}^{25} W'(t)\,dt$

(given)

$W(25) = \int_{20}^{25} W'(t)\,dt + 71 = 73.043$

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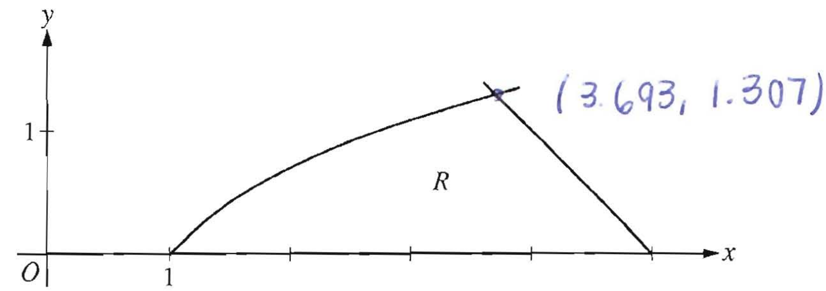

2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

$$\int_{1}^{3.693} \ln(x)\,dx + \int_{3.693}^{5} (5-x)\,dx$$

$$= 2.986 \text{ u}^2$$

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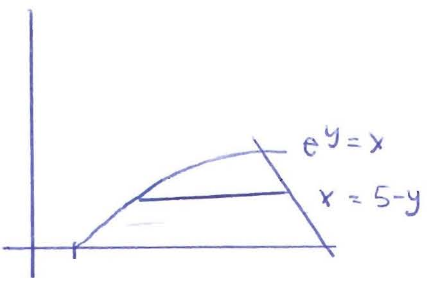
Continue problem 2 on page 7.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$\int_{1}^{3.693} (\ln(x))^2 dx + \int_{3.693}^{5} (5-x)^2 dx$$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.



$$\int_{0}^{k} 5-y-e^y \, dy = \frac{1}{2} * 2.986$$

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CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

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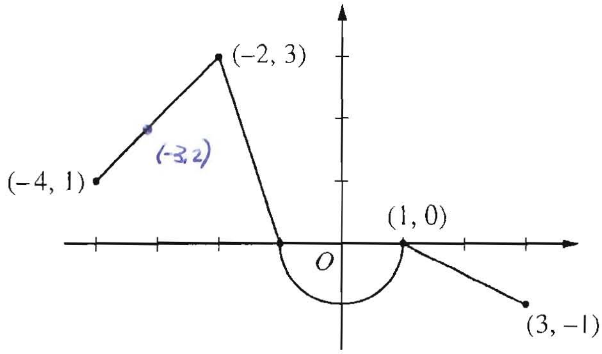

Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

$g(2) = \int_1^2 f(t)dt = \cancel{\tfrac{1}{2}(2)}(-1) = -1\ u^2$

$g(-2) = \int_1^{-2} f(t)dt = -\int_{-2}^{1} f(t)dt = \frac{1}{2}(1)(3) - \frac{\pi(1)^2}{2}$

$= -\left[\frac{3}{2} - \frac{\pi}{2}\right] = \pi/2 - 3/2$ or $\frac{\pi - 3}{2}$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(x) = \frac{d}{dx}\int_1^x f(t)dt = f(x) \rightarrow g'(-3) = f(-3) = 2$

$\frac{1-3}{-4+2} = \frac{-2}{-2} = 1$

$g''(x) = f'(x) \longrightarrow g''(-3) = f'(-3) = 1$

Continue problem 3 on page 15.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

Hori. Tangent → derivative = 0 $g'(x) = f(x)$

$f(x) = 0$

$x = -1$ ⟶ switches from + to − (maximum)

$x = 1$ ⟶ neither. slope does not change sign.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

POI : 2nd derivative = 0

$g''(x) = f'(x) = 0$ $g''(x) > 0 \rightarrow g''(x) < 0$

or

$g''(x) < 0 \rightarrow g''(x) > 0$

$x = -2$, $x = 0$, $x = 1$

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

$f(x) = (25 - x^2)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(25 - x^2)^{-1/2} * (-\cancel{2}x)$

$= \dfrac{-x}{\sqrt{25 - x^2}}$

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

$x = -3 \longrightarrow y = \sqrt{25 - 9} = 4$

$(-3, 4)$

$m = \dfrac{-(-3)}{4} = \dfrac{3}{4}$

$y - 4 = \frac{3}{4}(x + 3)$

$y = \frac{3}{4}(x + 3) + 4$

Continue problem 4 on page 17.

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NO CALCULATOR ALLOWED

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

A function is continuous if $\lim\limits_{x \to a^+} = \lim\limits_{x \to a^-} = \lim\limits_{x \to a}$

$\lim\limits_{x \to -3^-} f(x) = \lim\limits_{x \to 3^-} \sqrt{25 - x^2} = 4$

$\lim\limits_{x \to 3^+} x + 7 = 4$

$g(-3) = f(-3) = 4$

The function is continuous.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

$25 \times 5 = 125$

$u = 25 - x^2$ $\quad x = 0 \rightarrow u = 25$

$du = 2x\, dx$ $\quad x = 5 \rightarrow u = 0$

$\frac{1}{2} du = x\, dx$

$\frac{1}{2} \int_{25}^{0} u^{1/2} du = \frac{-1}{2} \int_0^{25} u^{1/2}\, du = \frac{-1}{2} \left[\frac{2}{3} u^{3/2} \right]_0^{25}$

$= \frac{-1}{2} \left[\frac{2}{3}(125) - 0 \right]$

$= \frac{-250}{6} = \frac{-125}{3} \; u^2$

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5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

$B = 40$

$$\frac{dB}{dt} = \frac{1}{5}(100-40)$$
$$= \frac{60}{5} = 12$$

$B = 70$

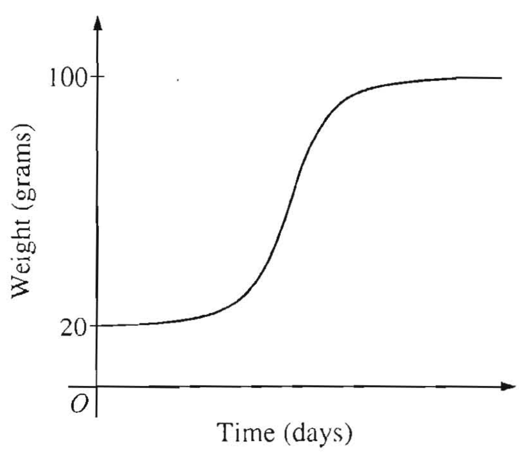
$$\frac{dB}{dt} = \frac{1}{5}(100-70)$$
$$= \frac{30}{5} = 6$$

The bird is gaining weight when it weighs 40 grams because the slope (denvative/rate) is larger.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

$20 - \frac{1}{5}B$

$$\frac{d^2B}{dt^2} = \frac{-1}{5}\frac{dB}{dt}$$
$$= \frac{-1}{5}\left(20 - \frac{1}{5}B\right)$$
$$= -4 + \frac{1}{25}B$$



The 2ind derivative is negative
↳ concave down
Part of the graph is concave up.

$$-4 + \frac{1}{25}B = 0$$

Continue problem 5 on page 19.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\int \frac{dB}{100-B} = \int \frac{1}{5}\,dt$$

$$-\ln|100-B| = \frac{1}{5}t + C \qquad -\ln|100-20| = C$$

$$C = -\ln|80|$$

$$e^{\ln|100-B|} = e^{-\frac{1}{5}t+C}$$

$$100 - B = e^{-1/5+C}$$

$$B = Ce^{-1/5t} + 100$$

$$B = -80e^{-1/5t} + 100$$

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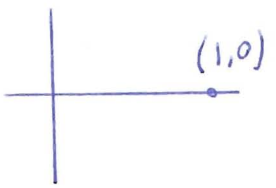
6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

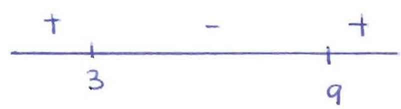
(a) For $0 \le t \le 12$, when is the particle moving to the left?

↳ velocity is negative

$\cos(\pi/6\, t) = 0$ $\qquad \pi/6\, t = 1$ $\qquad t = 3, 9$

$\pi/6\, t = \cancel{\cos^{-1}(0)}\ 1$





$3 < x < 9$

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

$\int_0^6 |\cos(\pi/6\, t)|\, dt$

Continue problem 6 on page 21.

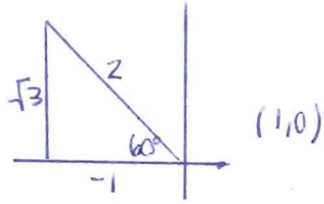
NO CALCULATOR ALLOWED

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$v(t) = \cos\left(\frac{\pi}{6}t\right)$

$a(t) = -\sin\left(\frac{\pi}{6}t\right) * \frac{\pi}{6}$

$v(4) = \cos(2\pi/3) = -1/\sqrt{3}$ (negative)



Because the velocity and acceleration have the same sign (negative) the speed is increasing at $t=4$.

$a(4) = -\sin\left(\frac{2\pi}{3}\right) * \frac{\pi}{6} = -\frac{\sqrt{3}}{2} * \frac{\pi}{6}$ (neg.)

(d) Find the position of the particle at time $t = 4$.

$\int_0^4 \cos\left(\frac{\pi}{6}t\right)dt - 2$

↑initial position

$u = \pi/6\, t$

$du = \pi/6\, dt$

$\frac{6}{\pi} du = dt$

$\int \cos(u)du$

$= \frac{6}{\pi}\left[\sin(\pi/6\, t)\right]_0^4 - 2$

$= \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - \sin(0)\right] - 2$

$= \frac{6}{\pi}\left[\frac{\sqrt{3}}{2} - 0\right] - 2$

$\frac{3\sqrt{3}}{\pi} - \frac{2\pi}{\pi}$

$\frac{3\sqrt{3} - 2\pi}{\pi}$

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