

Name \_\_\_\_\_ Date \_\_\_\_\_  
 PreCalculus Final Review

1. Which expression always equals 1?

- (1)  $\cos^2 x - \sin^2 x$
- (2)  $\cos^2 x + \sin^2 x$
- (3)  $\cos x - \sin x$
- (4)  $\cos x + \sin x$

2. The expression  $\sin 2A + \cos A$  is equivalent to

- (1)  $\cos A(2 \sin A + 1)$
  - (2)  $\cos A(\cos A + 1)$
  - (3)  $2(\sin A + \cos A)$
  - (4)  $\cos A(\sin A + 1)$
- $2 \sin A \cos A + \cos A$   
 $\cos A(2 \sin A + 1)$

3. The expression  $\cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ$  is equivalent to

- (1)  $\cos 30^\circ$
  - (2)  $\cos 50^\circ$
  - (3)  $\sin 30^\circ$
  - (4)  $\sin 50^\circ$
- $\cos(40 - 10)$

4. If  $x$  is an acute angle, and  $\cos x = \frac{4}{5}$ , then  $\cos 2x$  is equal to

$$\begin{aligned} & \downarrow \\ & 2 \cos^2 x - 1 \\ & 2\left(\frac{4}{5}\right)^2 - 1 \\ & = \boxed{\frac{7}{25}} \end{aligned}$$

5. If  $x$  is an acute angle and  $\sin x = \frac{12}{13}$ , then  $\cos 2x$  equals

$$\begin{aligned} & \downarrow \\ & 1 - 2 \sin^2 x \\ & 1 - 2\left(\frac{12}{13}\right)^2 = \boxed{-\frac{119}{169}} \end{aligned}$$

6. What is the exact value of  $\csc 60^\circ$ ?

$$\frac{1}{\sin 60} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

7. Simplify the expression  $\frac{\sin 2\theta}{2 \cos \theta} = \frac{2 \sin \theta \cos \theta}{2 \cos \theta} = \boxed{\sin \theta}$

8. Find all values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  that satisfies the equation  $\cos x - \sin 2x = 0$

$$\begin{aligned} & \cos x - 2 \sin x \cos x = 0 \\ & \cos x(1 - 2 \sin x) = 0 \\ & \cos x = 0 \quad \sin x = \frac{1}{2} \\ & \boxed{90, 270} \quad \boxed{30, 150} \end{aligned}$$

9. Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  that satisfy the equation  $2\sin^2 \theta = 1 + \sin \theta$ .

$$2\sin^2 \theta - \sin \theta - 1 = 0$$

$$(2\sin \theta + 1)(\sin \theta - 1) = 0$$

$$\sin \theta = -\frac{1}{2} \quad \sin \theta = 1$$

$$\boxed{\theta = 210^\circ, 330^\circ} \quad \boxed{\theta = 90^\circ}$$

10. Express as a single fraction the exact value of  $\cos 75^\circ$

$$\cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

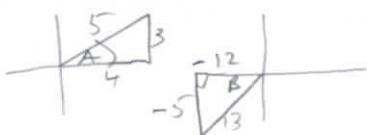
$$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

11. Solve:  $\frac{\sin^2 \theta}{1 + \cos \theta} = 1$  for all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$ .

$$\begin{aligned} \sin^2 \theta &= 1 + \cos \theta \\ 1 - \cos^2 \theta &= 1 + \cos \theta \\ \cos^2 \theta + \cos \theta &= 0 \end{aligned}$$

$$\begin{aligned} \cos \theta (\cos \theta + 1) &= 0 \\ \cos \theta = 0 &\quad \cos \theta = -1 \\ \boxed{\theta = 90^\circ, 270^\circ} &\quad \boxed{\theta = 180^\circ} \end{aligned}$$

12. If  $\cos A = \frac{4}{5}$ ,  $\sin B = -\frac{5}{13}$ , and angle A is in Quadrant I and angle B is in Quadrant III what is the value of  $\cos(A+B)$ ?



$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{5}{13}\right) &= \boxed{-\frac{33}{65}} \end{aligned}$$

13. Prove:  $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

$$\frac{2\sin x \cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x}$$

$$\frac{2\cos x - \frac{2\cos^2 x - 1}{\cos x}}{\cos x}$$

$$\begin{aligned} \frac{2\cos^2 x}{\cos x} - \frac{2\cos^2 x - 1}{\cos x} &= \frac{1}{\cos x} = \sec x \\ \sec x &= \sec x \checkmark \end{aligned}$$

14. Solve  $\sqrt{3}\sin x + 3\cos x = 0$

$$\begin{aligned} \frac{\sqrt{3}\sin x}{\cos x} - \frac{3\cos x}{\cos x} &= \tan x = -\frac{3}{\sqrt{3}} \\ \sqrt{3}\tan x = -3 & \end{aligned}$$

$$\boxed{x = 120^\circ, 300^\circ}$$

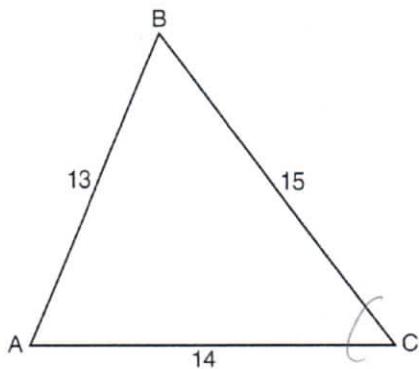
15. In  $\triangle ABC$ ,  $a = 15$ ,  $b = 14$ , and  $c = 13$ , as shown in the diagram below.

What is the  $m\angle C$ , to the nearest degree?

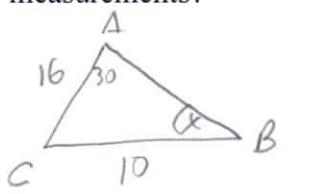
$$13^2 = 14^2 + 15^2 - 2(14)(15) \cos x$$

$$\cos x = .6$$

$$\boxed{x = 53}$$



16. In triangle ABC  $a = 10$ ,  $b = 16$ , and  $m\angle A = 30^\circ$ . How many distinct triangles can be drawn given these measurements?



$$\frac{10}{\sin 30} = \frac{16}{\sin x}$$

$$\sin x = .8$$

$$x = 53, 126$$

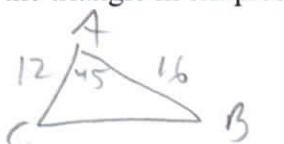
2 triangles

17. Find the sum of  $\sin 120^\circ$  and  $\tan 240^\circ$  expressed as a single fraction in simplest radical form

$$\sin 60 + \tan 60$$

$$\frac{\sqrt{3}}{2} + \sqrt{3} = \boxed{\frac{3\sqrt{3}}{2}}$$

18. In  $\triangle ABC$ , two sides measure 12 feet and 16 feet and the included angle is  $45^\circ$ . What is the *exact* area of the triangle in simplest form?



$$A = \frac{1}{2}(12)(16)\sin 45$$

$$A = \boxed{48\sqrt{2}}$$

19. Two forces of 25 newtons and 85 newtons acting on a body form an angle of  $55^\circ$ . Find the magnitude of the resultant force, to the *nearest hundredth of a newton*.

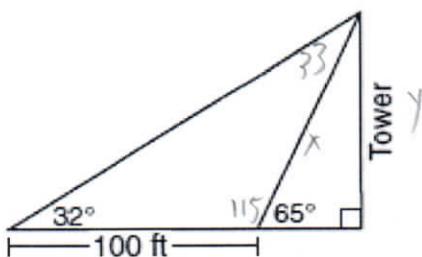


$$x^2 = 85^2 + 25^2 - 2(85)(25)\cos 55$$

$$x^2 = 10287$$

$$x = \boxed{101.43}$$

20. The accompanying diagram shows the plans for a cell-phone tower that is to be built near a busy highway. Find the height of the tower, to the *nearest foot*.



$$\frac{100}{\sin 33} = \frac{x}{\sin 32}$$

$$x = 97.3$$

$$\sin 65 = \frac{y}{97.3}$$

$$y = \boxed{88}$$

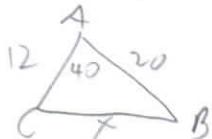
21. In triangle RST,  $m\angle R = 105^\circ$ ,  $r = 12$ , and  $t = 10$ . Find  $m\angle T$  to the nearest integer.



$$\frac{10}{\sin T} = \frac{12}{\sin 105^\circ}$$

$$x = 54$$

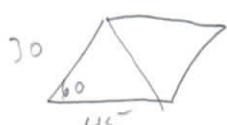
22. In triangle ABC, side  $b = 12$ , side  $c = 20$  and  $m\angle A = 40^\circ$ . Find side  $a$  to the nearest integer.



$$x^2 = 12^2 + 20^2 - 2(12)(20)\cos 40^\circ$$

$$x = 13$$

23. The lengths of two sides of a parallelogram are 30 and 45 centimeters. Their included angle measures  $60^\circ$ . Find the EXACT area of the parallelogram.



$$A = (30)(45)\sin 60^\circ$$

$$A = 675\sqrt{3}$$

24. Find the largest angle, to the nearest tenth of a degree, of a triangle whose sides are 9, 12 and 18.

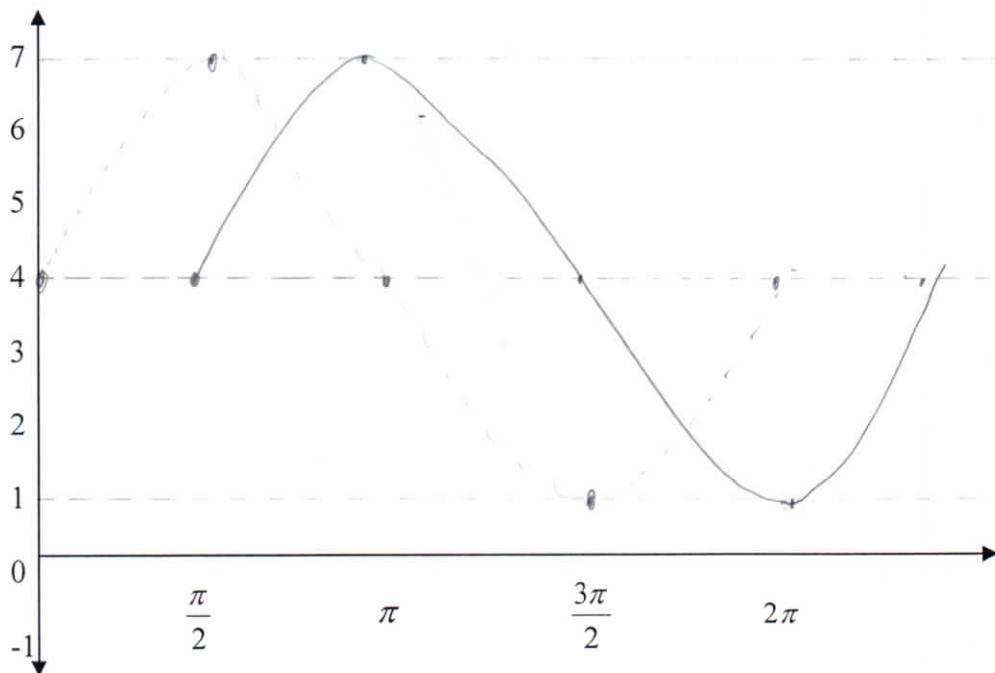


$$18^2 = 9^2 + 12^2 - 2(9)(12)\cos x$$

$$x = 117.3$$

25. a) Using a dotted line, graph  $y = 3\sin x + 4$  from  $0 \leq x \leq 2\pi$

- b) On the same axis, using a solid line, graph  $y = 3\sin(x - \frac{\pi}{2}) + 4$  from  $0 \leq x \leq 2\pi$



26. Solve all in  $0^\circ \leq x \leq 360^\circ$ :  $\cos^2 x + \cos 2x = \frac{5}{4}$

$$4\cos^2 x + 4\cos 2x = 5$$

$$4\cos^2 x + 4(2\cos^2 x - 1) = 5$$

$$4\cos^2 x + 8\cos^2 x - 4 - 5 = 0$$

$$12\cos^2 x - 9 = 0$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = 30, 150, 210, 330$$

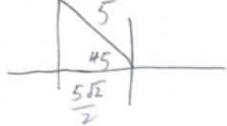
27. Solve for  $x$ :  $x^{\frac{2}{3}} = 25i$

$$x = (25i)^{\frac{1}{2}}$$

$$x = (25 \text{ cis } 90)^{\frac{1}{2}}$$

$x = 125 \text{ cis } 135$   
 $x = 125 \text{ cis } 315$

28. Represent the complex number graphically and give the rectangular form of the number:

$$5\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = 5 \text{ cis } 135 = \boxed{-\frac{5\sqrt{2}}{2} + \frac{5i\sqrt{2}}{2}}$$


29. Represent the complex number graphically and give the polar form of the number:  $-6i$

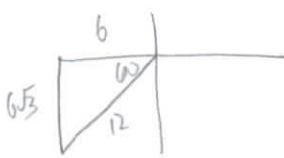


30. Solve all in  $0^\circ \leq x \leq 360^\circ$ :  $\sin x = \sin(2x)$

$$\begin{aligned} \sin x &= 2 \sin x \cos x \\ \sin x (1 - 2 \cos x) &= 0 \\ \sin x &= 0 \quad \cos x = \frac{1}{2} \end{aligned}$$

$x = 0, 60, 180, 300, 360$

31.  $(-6 - 6i\sqrt{3})^{\frac{1}{4}} = (12 \text{ cis } 240)^{\frac{1}{4}}$



$4\sqrt{2} \text{ cis } 60$   
 $4\sqrt{2} \text{ cis } 150$   
 $4\sqrt{2} \text{ cis } 240$   
 $4\sqrt{2} \text{ cis } 330$

For # 32-35, use  $z_1 = 9 \text{ cis } 210^\circ$  and  $z_2 = \frac{1}{3} \text{ cis } 30^\circ$

32.  $z_1 \cdot z_2$      $3 \text{ cis } 240$

33.  $z_1 \div z_2$      $27 \text{ cis } 180$

34.  $|z_1|$     9

35.  $\overline{z_2}$      $\frac{1}{3} \text{ cis } (-30)$  or  $\frac{1}{3} \text{ cis } 330$

36.  $X = (2 \text{ cis } 15^\circ)^6$      $64 \text{ cis } 90$

$$37. \lim_{x \rightarrow 4^+} \frac{\sqrt{x}}{x^2} = \frac{1}{8}$$

$$38. \lim_{x \rightarrow \infty} \sqrt[3]{\frac{8+x^2}{x(x+1)}} = 1$$

$$39. \lim_{x \rightarrow -\infty} \frac{x^8 - 4x^2 + 7}{3x^5 - x + 1} = -\infty$$

$$40. \lim_{x \rightarrow 3} \frac{x-3}{x^2 - 2x - 3} = \frac{1}{4}$$

$$41. \lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} = \frac{1}{2\sqrt{3}}$$

$$42. \lim_{x \rightarrow 1} \frac{\ln x + 3x}{x} = 3$$

$$43. \lim_{x \rightarrow 0} \frac{(3+x)^2 - 9}{x} = 6$$

$$44. \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = -\frac{1}{4}$$

$$45. \lim_{x \rightarrow 0} \left( \sqrt{x^2 + 9} - \frac{x^2 + 3x}{x} \right) = 0$$

$$46. \lim_{x \rightarrow \infty} \left( \frac{1}{x} - \frac{x}{x-1} \right) = -1$$

$$47. \lim_{x \rightarrow -1^+} \frac{x^2 - 2x}{x+1} = \infty$$

48. Find all points on the graph of  $y = x^3 - 3x$  where the tangent line is parallel to the x-axis.

$$\begin{aligned}y' &= 3x^2 - 3 \\3x^2 - 3 &= 0 \\x^2 &= 1 \\x &= \pm 1\end{aligned}$$

$$\boxed{(1, -2) \quad (-1, 2)}$$

Slope = 0

49. Find the coordinates of the point(s) on the graph of  $y = \frac{1}{x^2}$ , where the tangent line to the graph of  $y$  is parallel to the line  $2x + 8y = 5$ .

$$\begin{aligned}8y &= -2x + 5 \\y &= -\frac{1}{4}x + \frac{5}{8}\end{aligned}$$

$$y' = -2x^{-3}$$

$$\frac{-2}{x^3} = -\frac{1}{4}$$

$$x = 2$$

$$\boxed{(2, \frac{1}{4})}$$

50. Using the definition of derivative, find  $f'(x)$  given  $f(x) = x^2 - 4x + 1$ . Find the equation of the tangent line at  $x=2$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) - 1 - (x^2 - 4x + 1)}{h}$$

$$f(2) = -3$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h}$$

$$f'(2) = 0$$

$$\lim_{h \rightarrow 0} 2x + h - 4 = 2x - 4$$

$$y + 3 = 0(x - 2)$$

51. Find  $f'(x)$  and simplify given  $f(x) = \frac{5x}{\sqrt{x}} + \frac{4}{3x^3} = 5x^{\frac{1}{2}} + \frac{4}{3}x^{-3}$

$$\boxed{y = -3}$$

$$\boxed{f'(x) = \frac{5}{2}x^{-\frac{1}{2}} - 4x^{-4}}$$

52. Find  $f'(x)$  and DO NOT simplify given  $f(x) = (5x+1)(4\sqrt[3]{x} + e)$

$$f'(x) = 5(4\sqrt[3]{x} + e) + \frac{4}{3}x^{-\frac{2}{3}}(5x+1)$$

53. Find  $f'(x)$  and DO NOT simplify  $f(x) = \frac{4x^5 + 7}{\pi}$

$$f'(x) = \frac{20x^4}{\pi}$$

54.  $f(x) = \sqrt[4]{\left(\frac{2x-5}{5x+2}\right)}$

$$f'(x) = \frac{1}{4} \left(\frac{2x-5}{5x+2}\right)^{-\frac{3}{4}} \left( \frac{(5x+2)(2) - (2x-5)(5)}{(5x+2)^2} \right)$$

55. Let  $f(x) = \frac{x^2 - 1}{x^2 + 1}$

a) What is the slope of the graph at  $x = -2$ ?

$$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$f'(-2) = \frac{-8}{25}$$

b) Is the function value of  $f$  changing more rapidly at  $x = -2$  or  $x = -1$ ? Justify your answer

$$f'(-2) = -\frac{8}{25}$$

$$f'(-1) = -1 \rightarrow \text{more rapid}$$

c) When is the equation of the tangent line horizontal?

$$\text{Slope} = 0$$

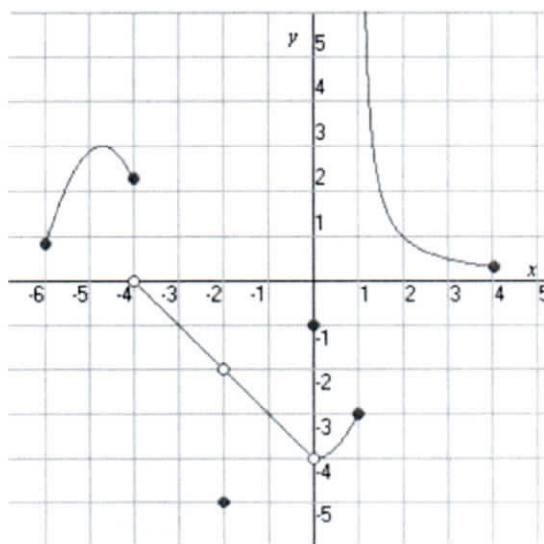
$$\frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = 0$$

$$2x^3 + 2x - 2x^3 + 2x = 0$$

$$4x = 0$$

$$\boxed{x=0}$$

56. Given the following graph below of  $f(x)$ , find the following:



(a)  $\lim_{x \rightarrow -4^-} f(x) = 2$

(b)  $\lim_{x \rightarrow -2} f(x) = -2$

(c)  $f(-2) = -5$

(d)  $\lim_{x \rightarrow 1^-} f(x) = 3$

(e)  $\lim_{x \rightarrow 1^+} f(x) = \infty$

(f)  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

(g)  $f(1) = -3$

(h)  $\lim_{x \rightarrow 2} f(x) = 1$